Mill's *System of Logic*

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Godden, D. (2014). Mill's *System of Logic*. In W.M. Mander (Ed.), *Oxford handbook of British philosophy in the nineteenth century* (pp. 44-70). Oxford: OUP.

ABSTRACT: This chapter situates Mill's *System of Logic* (1843/1872) in the context of some of the meta-logical themes and disputes characteristic of the 19th century as well as Mill's empiricism. Particularly, by placing the *Logic* in relation to Whately's (1827) *Elements of Logic* and Mill's response to the "great paradox" of the informativeness of syllogistic reasoning, the chapter explores the development of Mill's views on the foundation and function of, and the relation between, ratiocination and induction. It provides a survey of the Mill-Whewell debate on the nature of induction, Mill's account of putatively *a priori* disciplines such as the science of number, and Frege's criticisms of the *Logic* as psychologistic.

KEYWORDS: Frege, Gottolob; induction; logic, 19th century; Mill, J.S.; psychologism; ratiocination, theory of; *A System of Logic*; syllogism, theory of; Whately, Richard; Whewell, William.

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Introduction

John Stuart Mill's (1806-1873) *A System of Logic*, first published in 1843 and going through a total of eight editions over the next three decades, appeared at a pivotal juncture in the history of logic. Prior to the nineteenth century logic had suffered a great decline, coinciding with the scientific and metaphysical achievements of the enlightenment. Generally it was felt that logic, whose methods and systems of proof had made no advancement since Aristotle's syllogism, was incapable of advancing knowledge. By contrast, by the time of the *Logic*'s eighth edition in 1872, it was well known that the syllogism was incapable of representing the truth-preserving structure of many valid arguments. Moreover, while the conception of logic as a formal calculus can be traced back to Leibniz (1646-1716), significant advancements in the techniques for formalizing logical structure occurred in the time of Mill's *Logic*.[1] Yet, the *Logic* was – and remained – uninfluenced by these developments, holding fast to the idea that the syllogism embodied the form of all valid reasoning. Indeed, as Scarre notes (1989, p. 1), Mill, in a letter to John Elliot Cairnes (5.12.1871), complained that his contemporaries Jevons, Boole, De Morgan, and Hamilton "have a mania for encumbering questions with useless complications, and with a notation implying the existence of greater precision in the data than the questions admit of" (*CW* 17, pp. 1862-63). Rather than this "mania," Mill (*ibid*.) preferred that "scientific deductions should be made as simple and as easily intelligible as they can be made without ceasing to be scientific." Thus, the significance of the *System of Logic* should not be sought in its contribution to logical techniques or systems. Instead, its significance is best sought in a meta-logical context.

One way to approach the project of Mill's *Logic* is to understand it as an exposition of an empiricist account of the nature, structure, and foundations of inferential knowledge. Here at least one of Mill's targets was the rationalistic view that some substantive truths can be known by reason alone, and that *a priori* knowledge is required if truths deemed necessary can be known at all. This view Mill identified as perhaps the single greatest source of error in the natural as well as the moral sciences.

> The notion that truths external to the mind may be known by intuition or consciousness, independently of observation and experience, is, I am persuaded, in these times, the great intellectual support of false doctrines and bad institutions. By the aid of this theory, every inveterate belief and every intense feeling, of

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[1] George Bentham's (1800-1884) *Outline of a New System of Logic* (1827) proposed quantifying the predicates of categorical statements as well as their subjects. DeMorgan's (1806-1871) *Formal Logic* (1847) introduced a logic of relations, on which the copula of categorical statements is only one relation, and such that the validity of the *Barbara* syllogism, for example, is explained as a particular instance of the transitivity of a relation. Boole's (1815-1864) *Mathematical Analysis of Logic* (1847) offered an algebraic formalization of syllogistic logic, which was developed in his *Laws of Thought* (1854). Finally, in removing many of the mathematical elements from the Boolean symbolism, Jevons's (1835-1882) *Pure Logic* (1864) offered the logicist thesis that mathematics is logical in nature. Indeed 1879 would see the publication of Frege's (1848-1925) *Begriffschrift* which offered a fully articulated quantified predicate calculus.

For a developed discussion of these themes see Gray's "Some British logicians" (this volume). See also Gabbay & Woods (2004), Peckhaus (1999), Simons (2003), and Thiel (1982), to whom the preceding summary is indebted.

> which the origin is not remembered, is enabled to dispense with the obligation of justifying itself by reason, and is erected into its own all-sufficient voucher and justification. (*CW* 1, p. 233)

As Mill describes in his *Autobiography*, the *Logic* offers a reply to such theories by providing a detailed account of those domains of knowledge, whether abstract or scientific, founded solely on empiricist principles.

> In attempting to clear up the real nature of the evidence of mathematical and physical truths, the *System of Logic* met the intuition philosophers on ground on which they had previously been deemed unassailable; and gave its own explanation, from experience and association, of that peculiar character of what are called necessary truths, which is adduced as proof that their evidence must come from a deeper source than experience. (*ibid.*)

A second way to contextualize the *Logic* is to situate it in the development of Mill's meta-logical theories concerning the nature, and the epistemic foundation and utility of, demonstrative reasoning. The problem which captured Mill's attention here is that deduction, if valid, must be non-ampliative and hence is seemingly incapable of advancing knowledge.

> It is universally avowed that a syllogism is vicious [i.e., invalid] if there be anything more in the conclusion than was assumed in the premises. But this is, in fact, to say that nothing ever was, or can be, proved by syllogism, which was not known, or assumed to be known, before. (*CW* 7, p. 183)

In the *Logic*, Mill (*ibid*.) took issue both with those who, in the face of the above, inconsistently continued "to represent the syllogism as the correct analysis of what the mind actually performs in discovering and proving the larger half of the truths, whether of science or daily life, which we believe," and again with those who, as a consequence of the above view, were "led to impute uselessness and frivolity to the syllogistic theory itself, on the ground of the *petitio principii* which they allege to be inherent in every syllogism." Mill sought to resolve this dilemma by presenting a corrected account of the relationship between syllogistic and inductive reasoning.

Epistemological and Metaphysical Background[2]

For Mill, "Every consistent scheme of philosophy requires as its starting-point, a theory respecting the sources of human knowledge, and the objects which the human faculties are capable of taking cognizance of" (*CW* 10, p. 125). Placing himself within the "School of Experience," Mill's theories on these points are empiricist in their epistemology, and phenomenalist in their metaphysics (cf. McRae 1973, p. xxii).

[2] For an extended overview of these topics, see Donner & Fumerton (2001), Fumerton (2009), and Skorupski (1994), to whom the discussion offered here is indebted.

Epistemically, Mill claimed, "all knowledge consists of generalizations from experience. … There is no knowledge *a priori*; no truths cognizable by the mind's inward light, and grounded on intuitive evidence. Sensation, and the mind's consciousness of its own acts, are not only the exclusive sources, but the sole materials of our knowledge" (*CW* 10, p. 125). This empiricism involves a standard internalist and foundationalist account of the structure of justification.

> Truths are known to us in two ways: some are known directly, and of themselves; some through the medium of other truths. The former are the subject of Intuition, or Consciousness; the latter, of Inference. The truths known by intuition are the original premises from which all others are inferred. Our assent to the conclusion being grounded on the truth of the premises, we could never arrive at any knowledge by reasoning, unless something could be known antecedently to all reasoning. (*CW* 7, pp. 6-7)

While the *Logic* treats of inference and thereby of derived truths, the original premises of inference are items of which we are directly conscious. These, for Mill, are objects of pure consciousness (e.g., sensations), from which we must infer truths about the external world.

> [O]f the outward world, we know and can know absolutely nothing, except the sensations which we experience from it. (*CW* 7, p. 62)
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> Of nature, or anything whatever external to ourselves, we know … nothing, except the facts which present themselves to our senses, and such other facts as may, by analogy, be inferred from these. (*CW* 10, p. 125)

Mill's empiricist epistemology readily gives rise to a phenomenalist metaphysics. "Existence, so far as Logic is concerned about it, has reference only to phenomena; to actual, or possible, states of external or internal consciousness in ourselves or others" (*CW* 7, p. 604). Yet, the existing world is not limited to the present contents of our collective consciousness. Unperceived things are also properly said to exist, though for Mill such claims assert only counterfactual conditionals to the effect that were we in the right circumstances we would have certain perceptual experiences. We must infer the existence of unperceived objects by linking their existence to some known (i.e., directly perceived) thing using an inductive law of succession of coexistence (*CW* 7, pp. 605-6). Objects, then, are really "permanent possibilities of sensation." "The existence, therefore, of a phenomenon, is but another word for its being perceived, or the inferred possibility of perceiving it" (*CW* 7, p. 605).

> I believe that Calcutta exists, though I do not perceive it, and that it would still exist if every percipient inhabitant were suddenly to leave the place, or be struck dead. But when I analyse the belief, all I find in it is, that were these events to take place, the Permanent Possibility of Sensation which I call Calcutta would still remain; that if I were suddenly transported to the banks of the Hoogly, I should still have the sensations which, if now present, would lead me to affirm that Calcutta exists here and now. We may infer, therefore, that both philosophers

> and the world at large, when they think of matter, conceive it really as a Permanent Possibility of Sensation. (*CW* 9, p. 184)

In attempting to set the sciences, including putative *a priori* sciences, on this foundation, Mill provided an account of all knowledge as *a posteriori*. Only in this way, Mill felt, could one rightly understand the actual inferential structure of knowledge and the proper function of the syllogism. To appreciate this point and to place Mill's *Logic* in the theoretical context in which it was developed, it is best to begin with a review of some standard empiricist critiques of syllogistic inference, and of Whately's defense of the syllogism and its epistemic utility.

Whately and the Revival of Logic in the British Tradition[3]

Up until the adoption of Whately's (1787-1863) *Elements of Logic* (1827), the standard logic text at Oxford was Aldrich's (1647-1710) *Artis logicae compendium* (1691). Beginning with Aristotle's definition of the syllogism as "Discourse such that something being asserted, something necessarily follows from it" (*Prior Analytics* 24b 18; *Topics* 100a 25-27), Aldrich conceived of logic as the art of reasoning – a body of prescriptive rules designed to prevent indistinctness in apprehension, falsity in judgement, and inconclusiveness in discourse (or inference). At the time, such a conception was quite standard. For example, the *Port Royal Logic* (1662) characterized logic as the "art of thinking," defining it as "the art of conducting reason well in knowing things, as much to instruct ourselves about them as to instruct others" (Arnauld & Nicole 1996, p. 23).

Yet by the seventeenth century, this scholastic conception of syllogistic logic as the art of proper reasoning had fallen into disrepute from criticisms by enlightenment humanists. While it was acknowledged that reasoning could be represented (or, perhaps more accurately, reconstructed) syllogistically, the idea that Aristotelian logic could advance knowledge was rejected. If the conclusions of valid syllogisms follow necessarily from their premises, then those conclusions cannot assert anything beyond what is already stated in their premises. Hence, syllogistic validity seems only explained as a kind of *petitio principii*, and syllogistic reasoning is thereby entirely incapable of producing or justifying new knowledge. Thus in the *Advancement of Learning* (1605), Bacon (1561-1626) wrote:

> For as water ascends no higher than the level of the first spring, so knowledge derived from Aristotle will at most arise no higher again than the knowledge of Aristotle. And therefore, though a scholar must have faith in his master, yet a man well instructed must judge for himself. (1900, p. 20; cf. van Evra, p. 6)

While our knowledge without Aristotle might be comparatively dwarfed, to see further one must judge for oneself, and in order to judge for oneself one requires new means of judgement and inference.

[3] For an extended discussion of this topic, see James W. Allard's "Early 19th Century Logic" (this volume). The discussion offered here is heavily indebted to van Evra (1984), and George & van Evra (2002).

More harshly, Locke (1632-1704), in the *Essay Concerning Human Understanding* (1690), ridiculed the idea that Aristotle's syllogism provides the only means for inferential knowledge.

> If syllogisms must be taken for the only proper instrument of reason and means of knowledge, it will follow, that before Aristotle there was not one man that did or could know anything by reason, and that since the invention of syllogisms there is not one of ten thousand that doth.
> But God has not been so sparing to men to make them barely two-legged creatures, and left it to Aristotle to make them rational. (IV.xvii.4; 1975, p. 671)

Rather, according to Locke, syllogistic reasoning is only subsequent to knowledge, rather than advancing it into the territory of the unknown.

> A man knows first, and then his is able to prove syllogistically. ... Syllogism, at best, is but the art of fencing with the little knowledge we have, without making any addition to it. (IV,xvii, 6; 1975, p. 679)[4]

In place of the syllogism, Locke recommended "native rustic reason," or a common-sense employment of our common-sense reasoning abilities.

> Native rustic reason … is likelier to open a way to, and add to the common stock of mankind, rather than any scholastic proceeding. … For beaten tracks lead these sort of cattle ...whose thoughts reach only to imitation, *non quo eundum est, sed quo itur* [not where we ought to go, but where we have been]. (IV.xvii.6-7; 1975, pp. 679-80).

Whately's *Elements of Logic*

Whately's *Elements of Logic* (1827) sought to rehabilitate the study and practice of logic from the "common-sense" theorists,[5] by challenging the idea at the center of the enlightenment critique that logic is merely *an* art of reasoning – that there are alternative, non-syllogistic ways of reasoning correctly. Rather, according to Whately, "in every instance in which we *reason* ... a certain process takes place in the mind, which is one and the same in all cases, provided it be correctly conducted" (1975, p. 18). This has

[4] Previously, Descartes (1596-1650) had drawn a similar conclusion, in his *Discourse on Method* (1637), finding that the syllogism does not contribute to learning or the advancement of knowledge by providing a theory of evidence, but rather by explaining what is already known.

> I observed with regard to logic that syllogisms and most of its other techniques are of less use for learning things than for explaining to others the things one already knows or even ... for speaking without judgement about matters of which one is ignorant. (1985, p. 119; cf. van Evra, p. 7)

[5] In the Preface (pp. xii-xiv) to the edition of the *Elements* which appeared in the *Encyclopaedia Metropolitana* edition studied by Mill, Whately characterizes the common sense championed by the critics of logic as "an exercise of the judgment unaided by any Art or system of rules; such as we must necessarily employ in numberless cases of daily occurrence" (as quoted in Mill *CW* 11, p. 6).

important consequences for logic as a normative and theoretical discipline. Whately wrote:

> Logic has usually been considered by these objectors as professing to furnish a *peculiar* method of reasoning, instead of a method of analyzing that mental process which must *invariably* take place in all correct reasoning. ... For Logic, which is, as it were, the Grammar of Reasoning, does not bring forward the regular Syllogism as a distinct mode of argumentation; but as the form to which *all* correct reasoning may be ultimately reduced; and which, consequently, serves the purpose (when we are employing Logic as an *art*) of a test to try the validity of any argument. (1975, pp. 11-12)

For Whately, just as it is a mistake to think that there can be intelligible but ungrammatical sentences, so is it a mistake to think that there can be correct but non-syllogistic inferences. In this way, logic is not merely the art of reasoning, but it is also the science on which the art is based. "Logic," begins the *Elements*, "in the most extensive sense which the name can with propriety be made to bear, may be considered as the Science, and also the Art, of Reasoning" (1975, p. 1). As the art of reasoning logic "investigates the principles on which argumentation is conducted, and furnishes the [practical] rules to secure the mind from error in its deductions" (*ibid*.), while as a science logic involves "an analysis of the process of the mind in Reasoning" (*ibid*.).[6] [7]

Indeed, for Whately there is a single principle on which all reasoning is based: Aristotle's *dictum de omni et nullo*.

> All reasoning whatever, then, rests on the one simple principle laid down by Aristotle, that 'whatever is predicated, either affirmatively or negatively, of a term distributed, may be predicated in a like manner (i.e. affirmatively or negatively) of anything contained under that term.' (1975 p. 45; cf. pp. 31ff.)

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[6] Two points here merit comment. First, Whately's definition of logic as the art and science of reasoning seems not to reject or emend a scholastic account, but rather to return to it. Consider St. Thomas Aquinas's definition, offered in his commentaries on Aristotle's *Posterior Analytics*: "Logic is the science and art which directs the act of the reason, by which a man in the exercise of his reason is enabled to proceed without error, confusion, or unnecessary difficulty" (quoted in Hebermann et al, 1913, p. 324). Secondly, in taking logic to be the art and science of reasoning, Whately accepted the then unquestioned view that reasoning is the subject matter of logic. As Hume (1711-1776) had claimed in the *Treatise* (1739), "the sole end of logic is to explain the principles and operations of our reasoning faculty and the nature of our ideas" (1978, p. xv). Problematically, such a view readily leads to psychologism (see below).

[7] Competent reasoners need neither be (i) "conscious of this process [by which reasoning proceeds] in his own mind", nor (ii) "competent to explain the principles on which it proceeds" (1975, p. 18). Indeed, Whately agrees with Locke that people reasoned correctly before Aristotle described the valid forms of the syllogism. But this does not show that their reasoning was other than syllogistic, or that there are a variety of ways by which people may reason correctly. Rather, it shows that the practice of reasoning exists separately from the theory of it. For Whately, "the practice [of correct reasoning] not only may exist independently of the theory, but *must* have preceded the theory" (*ibid*.) since it is these processes which form the subject matter of the theory (i.e, the science of logic) itself. So, as McKerrow (Whately, 1975, p. viii) observes, by Whately's account Aristotle did not *invent* the syllogism, he merely discovered the principles of syllogistic reasoning and expressed them in the form of rules.

Whately on induction

Crucially then, Whately rejected induction as a legitimate form of reasoning. "Induction," he observed (1975, p. 208 ff.) ambiguously indicates two distinct activities: (i) the process of inquiry of collecting facts so as to obtain or evaluate premises for reasoning, and (ii) the process of inferring conclusions from those facts. Yet, as a process of inference, Whately claimed, induction is always an enthymematic deduction – a syllogism with the major premise suppressed – and thus not a uniquely legitimate form of reasoning. On the other hand, as a process of information acquisition, induction is not a form or reasoning at all, and thus not within the domain of logic. Rather, Whately (1975, p. 211) called this process *investigation*, and described it as the "*Inductive process*; which is that by which we gain, properly, *new truths*, and which is not connected with Logic." While investigation has epistemic functions such as knowledge acquisition and premise evaluation, it is not a part of logic.

Whately on the province of logic

Whately responded to the criticism that logic can make no contribution to the advancement of knowledge by considering the province of reasoning (1975, pp. 205 ff.). Logic is the study of reasoning as it applies to all subject matters, but it is not the task of logic to provide us with information concerning any particular subject matter. Thus logic is concerned with validity only (whether the conclusion *follows fairly from* the premises), not soundness (which includes whether the premises are *fairly laid down*) (1975, p. 210).

Whately admitted that there is such a thing as "logical discovery," whereby the mind becomes aware, through the process of demonstration, of knowledge to which it was previously entitled (being justified by knowledge consciously held), but of which it had not been explicitly aware prior to demonstration. Whately illustrated logical discovery by comparing it to a man discovering a vein of metal ore, of which he was previously unaware, running through his land. According to Whately, while the man already possessed the mineral deposits since they always belonged to him, for practical purposes they are new possessions to him since prior to his discovery of them he could make no use of them (1975, pp. 224-225). In this way, logic does indeed advance our knowledge, though it is incapable of discovering new truths which is the province of investigation not reasoning.

Mill's Early Work on Logic and Reception of Whately

Mill's early readings in logic

Not only did Whately's *Elements* provide the basis for Mill's first published work on logic, but it also inspired his *System of Logic* and informed many of its basic views. Mill's logical upbringing, as with all his early education, occurred at the direction and frequently at the hands of his father. At the age of twelve, Mill tells us in his *Autobiography*, his father had him read Aristotle's *Organon*, as well as several seventeenth century scholastic logic texts, and Hobbes's "Computatio sive Logica."

These texts became the subject of extended and intensive discussion during their walks together, where, Mill tells us,

> It was his [father's] invariable practice, whatever studies he exacted from me, to make me as far as possible understand and feel the utility of them: and this he deemed peculiarly fitting in the case of the syllogistic logic, the usefulness of which had been impugned by so many writers of authority. (*CW* 1, pp. 21-23)

Of his resulting "early [and] practical familiarity with school logic," Mill reflected, "I know of nothing, in my education, to which I think myself more indebted for whatever capacity of thinking I have attained" (*ibid*.).

In 1825 Mill joined the "Society for Students of Mental Philosophy" whose meetings involved the careful reading and thorough, open discussion of a variety of systematic treatises on a wide range of topics. In 1827 they turned to scholastic logic.

> Our first text book was Aldrich, but being disgusted with its superficiality, we reprinted one of the most finished among the many manuals of the school logic, which my father, a great collector of such books, possessed, the *Manuductto ad Logicam* of the Jesuit Du Trieu. After finishing this, we took up Whately's *Logic*, then first republished from the *Encyclopaedia Metropolitana*, and finally the "Computatio sive Logica" of Hobbes. These books, dealt with in our manner, afforded a wide range for original metaphysical speculation: and most of what has been done in the First Book of my *System of Logic*, to rationalize and correct the principles and distinctions of the school logicians, and to improve the theory of the Import of Propositions, had its origin in these discussions ... From this time I formed the project of writing a book on Logic, though on a much humbler scale than the one I ultimately executed. (*CW* 1, p. 125)

Mill's review of Whately's Elements

Mill's first published work on logic was an extensive review of the *Elements* appearing in the *Westminster Review* in 1828. Although it took issue with Whately on several significant points – including Whately's realist and essentialist account of classification and definition, which conflicted with Mill's own nominalsim (*CW* 11, p. 20 ff.) – Mill's review is generally flattering. To begin, Mill found that Whately's "vindication of the utility of logic is conclusive: his explanation of its distinguishing character and peculiar objects, of the purposes to which it is and is not applicable, and the mode of its application leave scarcely anything to be desired" (*CW* 11, pp. 3-4).

Crucially, in 1828 Mill accepted many of Whately's central views. Principal among these was Mill's acceptance of the view that logic is the art and science of reasoning. Additionally, Mill accepted that all reasoning is properly syllogistic. Consequently, induction is not a form of reasoning at all.

> Syllogistic reasoning is not a *kind* of reasoning, for *all* correct reasoning is syllogistic: and to *reason by induction* is a recommendation which implies as thorough a

> misconception of the meaning of the two words, as if the advice were, to *observe by syllogism*. (*CW* 11, p. 15)

Hence the rules of the syllogism provide correct norms for all reasoning. Mill further accepted the *dictum de omni et nullo* as the universal principle of all reasoning (*CW* 11, p. 17).

And, in response to those who advocated the common-sense method of rational analysis and evaluation, Mill replied:

> When all is done which has here been supposed [by the common-sense theorist], the argument is actually reduced to a series of syllogisms: so that the all-sufficiency of common-sense amounts only to this, that, if the man of common-sense *makes use of the same means* which logic supplies, he may attain the same end. (*CW* 11, p. 10)[8] [9]

Yet, despite his sympathies for Whately's project and perspective, Mill was deeply puzzled about how logic could be informative. He found it a paradox as yet unexplained on philosophical principles

> that mankind may correctly apprehend and fully assent to a general proposition, yet remain for ages ignorant of myriads of truths which are embodied in it, and which, in fact, are but so many particular cases of that which, as a general truth, they have long known. (*CW* 11, p. 33-34)

Even in 1828, Mill seemed unconvinced by Whately's explanation of logic's informativeness as "logical discovery:" an account which assigns to logic a merely cognitive – rather than a properly epistemic – function.

The *System of Logic*

The great paradox

It was not until the early 1830's, when he (*CW* 1, p. 189) returned to the "great paradox of the discovery of new truths by general reasoning," that Mill came to feel that Whately had explained away, rather than explained, the epistemic function of logic. While agreeing that "all reasoning was resolvable into syllogisms…," Mill further noted that "in

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[8] This reply echoes Whately's observation that "each [man] gives the preference to unassisted Common-Sense only in those cases where he himself has nothing else to trust to, and invariably resorts to the rules of art wherever he possesses the knowledge of them" (quoted in Mill *CW* 11, p. 7).

[9] Similarly, while Mill recognized that syllogistic analysis will not aid in the evaluation of arguments whose faults lie in their premises (*CW* 11, pp. 30-31), he found that syllogistic analysis seems to render many of the formal fallacies transparently invalid. "[W]hat higher compliment can be paid to the doctrine of the syllogism," he wrote (*CW* 11, p. 11), "than to say that the same fallacy, in the form of a syllogism, deceives nobody, which [quoting Dugald Stewart's (1753-1828) *Elements of the Philosophy of the Human Mind* (1792/1827)] 'may deceive half the world if *diluted* in a quarto volume.'"

every syllogism the conclusion is actually contained and implied in the premises." The paradox involves not only the logical problem of how conclusions, being contained in their premises, could express any new content or truth, but also the cognitive mystery of how "the theorems of geometry, so different to all appearance from, the definitions and axioms, could be all contained in these [definitions and axioms]" (*ibid*.).

It was in rereading Dugald Stewart's (1753-1828) *Elements of the Philosophy of the Human Mind* (Vol II, 1814) that Mill found the catalyst for his resolution of this "great paradox." There, as Mill tells us in his *Autobiography*,

> I came upon an idea of his [Stewart's] respecting the use of axioms in ratiocination, which I did not remember to have before noticed, but which now, in meditating on it, seemed to me not only true of axioms, but of all general propositions whatever, and to be the key of the whole perplexity. From this germ grew the theory of the Syllogism propounded in the second Book of the *Logic*; which I immediately fixed by writing it out. (*CW* 1, pp. 189-191)

While Mill does not reveal what specific idea in Stewart inspired his treatment of ratiocination in the *Logic*, we might surmise that it was Stewart's views about the probative role of general axioms in demonstrative reasoning.

> It was long ago remarked by Locke, of the axioms of geometry, as stated by Euclid, that although the proposition be at first enunciated in *general* terms, and afterwards appealed to, in its *particular* applications, as a principle *previously* examined and admitted, yet that the truth is not less evident in the latter case than in the former. He observes farther, that it is in some of its particular applications, that the truth of every axiom is originally perceived by the mind, and, therefore, that the general proposition, so far from being the *ground* of our assent to the truths which it comprehends, is only a verbal generalization of what, in particular instances, has already been acknowledged as true. (Stewart 1821, p. 23 [Vol.II, Ch.i, §1])

Accepting this picture effectively reverses the epistemic priority of general claims and the particular claims which are deduced from them. The growth of knowledge occurs not in the ratiocinative process of deriving theorems from axioms, but rather in establishing the axioms (i.e., general premises and principles of ratiocination) themselves. And this, consistently with Mill's empiricism, can only happen through experience of the particulars. Stewart wrote:

> [I]n all our reasonings about the established order of the universe, experience is our sole guide, and knowledge is to be acquired only by ascending from particulars to generals; whereas the syllogism leads us invariably from universals to particulars, the truth of which instead of being a *consequence* of the universal proposition, is *implied* and *presupposed* in the very terms of its enunciation. The syllogistic art, therefore, it has been justly concluded, can be of no use in extending our knowledge of nature. (Stewart 1821, p. 154 [Vol.II, Ch.iii, §2])

These ideas bear a remarkable resemblance to key elements of the theory of the syllogism found in Mill's *Logic*, and stand in stark contrast to the picture Mill accepted from Whately in 1828. Although the *Logic* retains Whately's general definition of logic as the art and science of reasoning, its views on the foundation and utility of, and relationship between, ratiocination and induction are dramatically changed. Siding with Stewart rather than Whately, the *Logic* explains the epistemic foundation of syllogistic reasoning as irreducibly based on the cogent induction of general premises, and the function of the syllogism as evaluative, rather than representative or epistemic.

Real versus apparent inference

To understand Mill's theory of ratiocination in the *Logic*, it is best to begin with his distinctions between real and apparent inference and real versus verbal propositions. For Mill, the requirement that logic be capable of advancing knowledge is a demand that logic should have an epistemic function – it must contain real inferences "in which we set out from known truths, to arrive at others really distinct from them" (*CW* 7, p. 162). Real inferences, for Mill, are ampliative – their conclusions assert more information than what is contained in their premises.

By contrast, an apparent inference "occurs when the proposition ostensibly inferred from another, appears on analysis to be merely a repetition of the same, or part of the same, assertion, which was contained in the first" (*CW* 7, p. 158).

> In all these cases there is not really any inference; there is in the conclusion no new truth, nothing but what was already asserted in the premises, and obvious to whoever apprehends them. The fact asserted in the conclusion is either the very same fact, or part of the fact, asserted in the original proposition. (*CW* 7, p. 160)

Corresponding to the distinction between real and apparent inference is Mill's distinction between real and verbal propositions – a distinction which Mill takes to correspond to Kant's distinction between synthetic and analytic judgements (*CW* 7, p. 116 fn.). Verbal propositions are ones "in which the predicate is of the essence of the subject (that is, in which the predicate connotes the whole or part of what the subject connotes, but nothing besides) answer no purpose but that of unfolding the whole or some part of the meaning of the name, to those who did not previously know it" (*CW* 7, p. 113). Technically, verbal propositions are not truth-apt: they are "not, strictly speaking, susceptible of truth or falsity, but only of conformity or disconformity to usage or convention; and all the proof they are capable of, is proof of usage" (*CW* 7, p. 109). Real propositions, by contrast,

> predicate of a thing some fact not involved in the signification of the name by which the proposition speaks of it; some attribute not connoted by the name. Such are ... all general or particular propositions in which the predicate connotes any attribute not connoted by the subject. All these, if true, add to our knowledge: they convey information, not already involved in the names employed. (*CW* 7, pp. 115-116)

The upshot of this picture is that the sciences, including putatively *a priori* sciences such as logic and mathematics, must contain real propositions if they are to be at all informative or true. Furthermore, if they are capable of advancing knowledge, they must involve real, rather than merely apparent, inferences whose conclusions are real propositions. Yet deduction, whose validity is explained in terms of its monotinicity, is clearly a case of merely apparent inference. Thus, if knowledge can be advanced through inference not only must the role of deduction be reconceived, but there must be another kind of inference capable of doing the required the epistemic lifting.

Mill's Account of Ratiocinative Logic

Ratiocination, for Mill, is "all reasoning by which, from general propositions previously admitted, other propositions equally or less general are inferred" (*CW* 7, p. 166).

Mill held, albeit mistakenly, that all valid ratiocination can be represented syllogistically, and that by a series of merely verbal transformations of the propositions comprising them, any syllogism can be represented in the first figure. Indeed, Mill claimed that all valid syllogisms can ultimately be represented as one of the following four: Affirmative syllogisms as either *Barbara* or *Darii*, and negative syllogisms as either *Clarent* or *Ferio* (*CW* 7, p. 168).

$$\begin{array}{lcl} \text{Affirmative} & \qquad & \text{Negative} \\ \text{Every B is C} & & \text{No B is C} \\ \left.\begin{array}{l}\text{All A}\\ \text{Some A}\end{array}\right\} \text{ is B,} & & \left.\begin{array}{l}\text{All A}\\ \text{Some A}\end{array}\right\} \text{ is B,} \\ \quad\text{therefore,} & & \quad\text{therefore,} \\ \left.\begin{array}{l}\text{All A}\\ \text{Some A}\end{array}\right\} \text{ is C.} & & \left.\begin{array}{l}\text{No A is}\\ \text{Some A is not}\end{array}\right\} \text{ C} \end{array}$$

Mill on the fundamental principle of ratiocination

By the time of the *Logic* Mill had abandoned the Whatelian *dictum de omni et nullo* as the fundamental principle of the syllogism, rejecting both realist (Platonist) and conceptualist (conventionalist) accounts of it. On a realist view (whereby universals are substances or real, abstract entities), the *dictum* is a real proposition – a "fundamental law of the universe" expressing the "intercommunity of nature" (*CW* 7, p. 174). Yet accepting that universals are not the objects of our experience, they are not real, and such a view is thus inconsistent with Mill's empiricism. By contrast, on a nominalist account (whereby universals are nothing more than collections of particular objects), consistent with his own empiricism, the *dictum* "merely amounts to the identical [i.e., verbal] proposition, that whatever is true of certain objects is true of each of those objects" (*CW* 7, p. 175). It thus becomes not an axiom but a definition explaining "in a circuitous and paraphrastic manner, the meaning of the word *class*" (*CW* 7, p. 175).

For Mill, since ratiocination is a mode of acquiring real knowledge, ratiocinative reasoning must be comprised of real, and not merely verbal, propositions. The terms of a syllogism's major premise are connotative names, denoting objects and connoting attributes. Thus the premise asserts of the two attributes connoted that they either always or never coexist together (*CW* 7, p. 177).

As such, Mill held the fundamental principle of ratiocination is the transitivity of coexistence. The validity of affirmative syllogisms is based on the principle that "things which coexist with the same thing coexist with one another" while valid negative syllogisms are underwritten by the principle that "a thing which coexists with another thing, with which other a third thing does not coexist, is not coexistent with that third thing" (*CW* 7, p. 178). Importantly for Mill, these principles are real and have the character of universally true laws of nature.

> These axioms manifestly relate to facts, and not to conventions; and one or the other of them is the ground of the legitimacy of every argument in which facts and not conventions are the matter treated of. (*CW* 7, p. 178)

Resolving the great paradox

While it is Mill's position, then, that valid ratiocination both involves, and relies upon, real propositions, the question remains as to whether ratiocination constitutes real inference or whether it is merely apparent. While Mill granted as "irrefragable" the view that "no reasoning from general to particulars can … prove anything: since from a general principle we cannot infer any particulars, but those which the principle itself assumes known," (*CW* 7, p. 184), he rejected both the common-sense theorist's conclusion that syllogistic reasoning is therefore useless or frivolous and Whately's view that the syllogism is "the correct analysis of what the mind actually performs in discovering or proving [the majority of our beliefs]" (*CW* 7, p. 183). Here, indeed, Mill finally rejected Whately's explanation of the epistemic function of ratiocination as "logical discovery" whereby entire sciences like geometry, "can be 'wrapt up' in a few definitions and axioms" (*CW* 7, p. 185).

The key to Mill's resolution, as inspired by Stewart, is found in his explanation of the epistemic foundation of general propositions, and hence of the epistemic status of a syllogism's major premise. Mill thought that the only view consistent with his own empiricism is that we come to know generalities through knowing their particulars: "[a]ll experience begins with individual cases, and proceeds from them to generals" (*CW* 7, p. 163). When discussing the justification of the major premise in a syllogism, Mill wrote:

> [W]hence do we derive our knowledge of that general truth? Of course from observation. Now, all which man can observe are individual cases. From these all general truths must be drawn, and into these they may again be resolved; for a general truth is but an aggregate of particular truths; a comprehensive expression, by which an indefinite number of individual facts are affirmed or denied at once. (*CW* 7, p. 186)

This view embraces Stewart's point that, while the truth of a particular proposition may, logically speaking, be a consequence of the truth of a general proposition which encompasses it, it is not an epistemic consequence. Rather, epistemically speaking, the truth of the particular is presupposed in the assertion of the general and hence must be known prior to it.[10]

Because general truths are not known prior to the particulars they encompass, they are not justification-conferring in ratiocination. Instead, Mill characterized general claims "registers of inferences already made." Because of this, ratiocination is not a form of real inference and by itself it is incapable of advancing knowledge even in Whately's cognitive sense of "logical discovery." Rather, ratiocination is itself dependent on another kind of inference whereby the truth of its major premises – indeed all general truths – are known: namely, induction. And, it is only in this sense that ratiocination is a mode of acquiring real knowledge.

> All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulae for making more: The major premise of a syllogism, consequently, is a formula of this description: and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according to* the formula: the real logical antecedent, or premise, being the particular facts from which the general proposition was collected by induction. (*CW* 7, p. 193)

And thus Mill resolved the great paradox. In doing so, he accepted Stewart's view that general truths do not, properly speaking, supply any epistemic grounds in ratiocination. Rather, general truths supply formulae in accordance with which we may reason. But the proper grounds for such reasoning are to be found in the particular truths already known and presupposed by general claims. Furthermore, the epistemic validity and utility of ratiocination depends on the validity of induction through which knowledge of general claims is achieved.

The proper function of the syllogism

In concluding that ratiocination does not confer justification but instead depends on induction for its epistemic foundation and utility, Mill also abandoned the Whatelian view that the syllogism analyzes "that mental process which must invariably take place in all correct reasoning" (*op. cit.*). Indeed, the syllogism does not correctly represent inference at all.

> [T]hough there is always a process of reasoning or inference where a syllogism is used, the syllogism is not a correct analysis of that process of reasoning or inference; which is, on the contrary, (when not a mere inference from testimony) an inference from particulars to particulars. (*CW* 7, p. 196)

Rather, "the syllogism is not the form in which we necessarily reason, but a test of reasoning: a form into which we may translate any reasoning, with the effect of exposing all the points at which any unwarranted inference can have got in" (*CW* 9, p. 390).

[10] To object, even empiricists should grant that general claims may be known by means other than through knowing each particular instance, as for example one might come to know general truths by testimony or authority and thereby legitimately infer to the particulars.

> The value, therefore, of the syllogistic form, and of the rules for using it correctly, does not consist in their being the form and the rules according to which our reasonings are necessarily, or even usually, made; but in their furnishing us with a mode in which those reasonings may always be represented, and which is admirably calculated, if they are inconclusive, to bring their inconclusiveness to light. An induction from particulars to generals, followed by a syllogistic process from those generals to other particulars, is a form in which we may always state our reasonings if we please. It is not a form in which we *must* reason, but it is a form in which we *may* reason, and into which it is indispensable to throw our reasoning, when there is any doubt of its validity. (*CW* 7, p. 198)

The ground of induction

Since induction provides the only proper epistemic grounds for ratiocination, a central task for Mill's *Logic* is to articulate some adequate account of the ground of the cogency of inductive inference. Unsurprisingly, Mill found the uniformity principle [UP] – "that the course of nature is uniform; that the universe is governed by general laws" – to be "the warrant for all inferences from experience" and "the fundamental principle, or general axiom, of induction" (*CW* 7, pp. 306-7). For Mill, our acceptance of this "universal fact" is justifiable experientially and inductively.[11]

> And, if we consult the actual course of nature, we find that the assumption is warranted. The universe, so far as known to us, is so constituted, that whatever is true in any one case is true in all cases of a certain description; the only difficulty is, to find what description. (*CW* 7, p. 306)

Yet, despite being the warrant licensing all inductive inference, UP is "itself an instance of induction:" it is a general truth whose justification is reached inductively. Indeed, Mill tells us, it is one of the one of the last inductions we naturally make, and one which is "founded on" many lesser, but prior, generalizations.

> We should never have thought of affirming that all phenomena take place according to general laws, if we had not first arrived, in the case of a great multitude of phenomena, at some knowledge of the laws themselves; which could be done no otherwise than by induction. (*CW* 7, p. 307)

Thus, although UP is the "ultimate major premise of all induction," it is the source of cogency in any particular inductive inference only in the same sense that the major (i.e., universal) premise of any syllogism establishes its conclusion: "not contributing at all to prove it, but being a necessary condition of its being proved" (*CW* 7, p. 308).

[11] On this point, Scarre (1998, p. 117) justifiably complains: "Characteristically, he [Mill] also held that it is experience which *justifies* the belief in uniformity, and thus unwittingly exposed himself to the objection that he was proposing an inductive justification of the very principle which he takes to warrant our inductive practice."

Moreover, the rigorously articulated and analysed form of induction employed in science and studied by logicians is parasitic on a kind of "spontaneous" induction performed naturally by all human beings.

> Many of the uniformities existing among phenomena are so constant, and so open to observation, as to force themselves upon involuntary recognition. Some facts are so perpetually and familiarly accompanied by certain others, that mankind learnt, as children learn, to expect the one where they found the other, … It will appear, I think, … that there is no logical fallacy in this mode of proceeding; but we may see already that any other mode is rigorously impractical: since it is impossible to frame any scientific method of induction, or test of the correctness of inductions, unless on the hypothesis that some inductions deserving of reliance have already been made. (*CW* 7, pp. 318-19)

In the final analysis, then, Mill seems to have embraced a naïve naturalism about the primitive cogency of induction. Although an inferential, rather than a basic, mode of knowledge, induction is nevertheless a primitive or basic *source* of justification. Rather than to discover and articulate any deeper justificatory foundation of induction than induction itself, the proper task of the logician is to supply a set of criteria or tests which can be used to distinguish good from bad inductive inferences.

> Why is a single instance, in some cases, sufficient for a complete induction, while in others, myriads of concurring instances, without a single exception known or presumed, go such a very little way towards establishing an universal proposition? Whoever can answer this question knows more of the philosophy of logic than the wisest of the ancients, and has solved the problem of induction. (*CW* 7, p. 314)

Laws of causation as principles of induction

The answer to this question, Mill felt, is to be found in the laws of causation. The uniformity of nature occurs along axes of simultaneity and succession, and the law describing nature's successive uniformity Mill called the Law of Universal Causation: "The truth that every fact which has a beginning has a cause, is coextensive with human experience" (*CW* 7, p. 325).

Yet, Mill recognized that nature's uniformity was neither simple nor uniform. Rather, "what is called the uniform course of nature is, that it is itself a complex fact, compounded of all the separate uniformities which exist in respect to single phenomena" (*CW* 7, p. 315). Thus, the task of the logician is to discover those particular uniformities which obtain in nature, and to ascertain those conditions which give rise to the occurrence of some one of them rather than to any of the others. It is only by knowing these individual causes, and the laws which govern them, that the logician will ever be able to distinguish those cases in which a single instance is sufficient for a complete induction – because the causal law instantiated is known – from those where myriads of concurring instances go nowhere to establishing a universal proposition – because whatever causal laws are at work are unknown. Knowledge of the laws of causation, then, is the only solution to the problem of induction, according to Mill. Thus the principal task of the inductive logician is no different from that of the natural scientist – to ascertain and articulate those causal laws of the universe.

Mill's methods

Yet, causes, insofar as we ordinarily experience them, are complex and indistinct. To the end of identifying those aspects of our experience which possess genuine causal efficacy, Mill offered a series of Baconian methods of experimental inquiry, or canons of induction (*CW* 7, pp. 389 ff.). The *method of agreement* seeks to identify operative causes by varying antecedent conditions so as to isolate exactly those conditions which are common to (i.e., agree with) all cases in which the phenomenon in question (i.e., effect) obtains. The process is one of elimination since "Whatever circumstance can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation" (*CW* 7, p. 390). The *method of difference* proceeds by holding antecedent conditions constant save for some single feature, while seeking variety (i.e., difference) in the obtaining of the effect. Here, those invariant antecedent conditions are eliminated as causally ineffectual, while the cause is isolated using the principle that "Whatever antecedent condition cannot be excluded without preventing the phenomenon, is the cause, or a condition, of that phenomenon" (*CW* 7, p. 391). These first two methods can be combined in the *joint method of agreement and difference*, which Mill identified as his third canon. The final two methods rely on the results of previous inductions. In the *method of residues*, known chains of cause and effect are used to explain certain features of a complex event, leaving a residue to be explained. This residue serves to isolate a set of (remaining) antecedent conditions which are supposed to be the causes of the (remaining) effects. Finally there is the *method of concomitant variations*, which relies on the principle that correlation is a sign of causation. When two phenomena co-vary, either one is the cause of the other or they share some common cause.

Mill versus Whewell on induction[12]

One of the most significant debates to arise out of Mill's *Logic* was his debate with William Whewell (1794-1866) concerning the nature and methods of induction. Mill's progress on the *Logic* had stalled for several years following his resolution of the "great paradox." Having placed induction at the center of logic, and the discovery of the laws of causation at the center of induction, Mill felt that he required a better understanding of scientific methodology and examples of its practice. One place Mill found this was in Whewell's newly published *History of the Inductive Sciences* (vol. 1, 1837) (*CW* 1, p. 215). In the preface to the first edition of the *Logic*, Mill tells us that "without the aid derived from the facts and ideas contained in ... [the *History*], the corresponding portion of this work would probably not have been written" (*CW* 7, p. cxiii). Further, while Mill was rewriting the *Logic* prior to its first publication, Whewell's *Philosophy of the*

[12] For a detailed discussion of Whewell's philosophy of science, see Duchene's contribution of that title (this volume). For a detailed discussion of the Mill-Whewell debate see Ducasse (1951), Forster (2009), Snyder (1997), and Strong (1955), to whom, as well as Scarre (1998), the discussion offered here is indebted.

Inductive Sciences (vol.1, 1840) appeared, providing Mill with a representative of "the German, or *a priori* view of human knowledge, and of the knowing faculties" (*CW* 1, p. 231-33; cf. Scarre 1998, p. 115) against which he could present his own empiricist views. As Passmore later observed, "Whewell's existence saved Mill the trouble of inventing him" (1957, p. 18; cf. Snyder 1997, p. 161).

Holding a chair in moral philosophy at Cambridge (indeed, soon to become master of Trinity College), and being a preeminent historian of science, Whewell presented a formidable opponent for Mill. As Reverend Sydney Smith (1771-1845) once remarked of Whewell, "Science is his forte; omniscience his foible" (quoted in Scarre 1998, p. 115). Whewell's reply to Mill's criticisms, *Of Induction, with especial reference to Mr. J. Stuart Mill's System of Logic*, appeared in 1849, to which Mill responded with some extensive revisions to the third edition to his *Logic* in 1851. The discussion offered here will identify two points of their debate: first, Whewell's criticism of Mill's experimental methods, and second on Whewell's view that induction involves the *colligation* of facts.

Mill and Whewell agreed that the experimental techniques of the sciences, being best suited for discovering the laws of nature, provide the best available paradigms for inductive methods. Yet, Whewell criticized Mill's canons claiming that they did not accurately describe the experimental procedures actually employed in science (McRae *CW* 7, p. xlvi).

> Who will tell us which of the methods of inquiry those historically real and successful inquiries exemplify? Who will carry these formulae through the history of the sciences, as they have really grown up; and show us that these four methods have been operative in their formation; or that any light is thrown upon the steps of their progress by reference to these formulae? (Whewell 1860, pp. 263-64; as quoted by Mill *CW* 7, p. 430).

Because of this, Mill took Whewell to claim that the canons are not a vehicle of discovery or advancement of knowledge.

Mill's strategy in responding to these criticisms echoes Whately's response to the "common-sense" criticism that syllogistic inference does not represent the actual practices of ordinary reasoners. Firstly, just as the practice of ratiocination is, as Whately claimed, separate from, and prior to, the theory of the syllogism, so, Mill claimed, are experimental practices separate from and prior to the theory (i.e., canons) of induction. Secondly, theory is related to practice not descriptively, but normatively. The canons do not seek to represent the activity of inquiry, but rather to articulate a set of rules to which the practice must accord if it is to succeed. Contrary to Whately, though, these rules do not describe a procedure which must be followed, but instead supply a set of conditions in terms of which inductive practices can be reconstructed in evaluation.

> The business of inductive logic is to provide rules and models (such as the syllogism and its rules are for ratiocination) to which if inductive arguments conform, those arguments are conclusive, and not otherwise. This is what the four methods profess to be, and what I believe they are universally considered to be by experimental philosophers, who had practised all of them long before any one sought to reduce the practice to theory. ... In saying that no discoveries were ever made by the four

> methods, he [i.e., Whewell] affirms that none were ever made by observation and experiment; for assuredly if any were, it was by processes reducible to one of those methods. (*CW* 7, pp. 430-31)

The second point of contention between Mill and Whewell divides their philosophies more deeply. To whatever extent canons of induction are sufficient to reveal the secret machinery of nature, Mill steadfastly held that laws thereby discovered are not only universal, "coextensive with the entire field of successive phenomena" (*CW* 7, p. 325), but also objective, holding "independently of all considerations respecting the ultimate mode of production of phenomena, and of every other question regarding the nature of 'things in themselves'" (*CW* 7, p. 327). For Mill, the nature's regularities are "out there" and that is where we find them.

While Mill and Whewell agreed as to the nature of induction, they disagreed as to how it is achieved. Specifically, they disagreed about what, if anything, is contributed by the mind itself in the inductive process beyond what is found "out there" among the phenomena.

For Whewell, all knowledge involves a combination of facts and ideas, and as such the mind actively supplies essential components to knowledge. In induction this involves something Whewell called the *colligation* of facts – or the bringing together of particular facts under some general conception which unites them. Importantly, this general conception, necessary for every induction, is not in the phenomena, or facts, themselves; rather it is supplied by the mind.[13]

> The particular facts are not merely brought together, but there is a new element added to the combination by the very act of thought by which they are combined. There is a Conception of the mind introduced in the general proposition, which did not exist in any of the observed facts. ... The facts are known, but they are insulated and unconnected, till the discoverer supplies from his own store a principle of connexion. The pearls are there, but they will not hang together till someone provides the string. (Whewell 1858, pp. 72-73)

In induction, the colligatory step is followed by a generalizing step in which the colligated property is projected over the entire class of phenomena including the unobserved instances. This generalizing step permits predictions which can be tested in future observations to confirm the hypothesis (Snyder 1997, pp. 170 ff.).

Of Whewell's colligation, Mill tells us that he "would gladly transfer all that portion of [Whewell's] book into [his] own pages" (*CW* 7, p. 294) except that it is does not belong to the process of induction, properly understood. Rather, for Mill, colligation has a cognitive function in the description of facts – where facts may be organized, perhaps fictitiously, according to many different unifying principles – but not an epistemic

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[13] Snyder (1997, pp. 173 ff.) distinguishes a special kind of induction, discovery (or "discoverer's induction") where the uniting principle is not already apparent, and claims that it is only in discovery that the colligating principle must be newly supplied by the mind. Technically this is correct, since the generalizing principle might already be known. But, for Whewell this knowledge could have only been the result of some previous colligation – i.e., the generalizing conception is already known because it has previously been supplied by the mind.

function in their explanation or prediction – whose success depends on the truth of the principles being applied, i.e., their actually being "out there."

> A conception implies, and corresponds to, something conceived: and though the conception itself is not in the facts, but in our mind, yet if it is to convey any knowledge relating to them, it must be a conception *of* something which really is in the facts, some property which they actually possess, and which they would manifest to our senses, if our senses were able to take cognizance of it. (*CW* 7, p. 295).

Because induction, for Mill, properly applies only to explanation and prediction, Whewell's colligation plays no role in it (*CW* 7, p. 299). Specifically, the principles relied on in induction are not supplied by the mind, but rather are found "out there" among the phenomena.

> No one ever disputed that in order to reason about anything we must have a conception of it; or that when we include a multitude of things under a general expression, there is implied in the expression a conception of something common to those things. But it by no means follows that the conception is necessarily pre-existed, or constructed by the mind out of its own materials. If the facts are rightly classed under the conception, it is because there is in the facts themselves something of which the conception is itself a copy; and which if we cannot directly perceive, it is because of the limited power of our organs, and not because the things itself is not there. (*CW* 7, p. 296)

Mill on the Science of Number

In view of Mill's empiricism and his position that induction provides the epistemic foundation for ratiocination, his account of putatively *a priori* sciences merits attention, as it remains a site of controversy.

Mill considered the laws of mathematics (arithmetic, algebra, and geometry) to be real, rather than verbal, propositions, comprising the most general laws of nature, specifically concerning the relation of resemblance among phenomena (*CW* 7, p. 607). Number terms are connotative names, denoting physical phenomena (collections or aggregates of things) and connoting some physical property of those phenomena, specifically the "characteristic manner in which the agglomeration is made up of, and may be separated into, parts" (*CW* 7, pp. 610-11). Because of this, Mill, in a passage that echoes Whately,[14] claimed

> All numbers must be numbers of something: there are no such things as numbers in the abstract. *Ten* must mean ten bodies, or ten sounds, or ten beatings of the pulse. But though numbers must be numbers of something, they may be numbers

[14] Whately (1975, pp. 13-14), writes: "All numbers ... must be numbers of some things, whether coins, persons, measures or anything else; but to introduce into the science [of arithmetic] any notice of the things respecting which calculations are made, would be evidently irrelevant, and would destroy its scientific character: we proceed therefore with arbitrary signs representing numbers in the abstract."

> of anything. Propositions, therefore, concerning numbers, have the remarkable peculiarity that they are propositions concerning all things whatever; all objects, all existences of every kind, known to our experience. (*CW* 7, p. 255).

As such, Mill held that physical facts are asserted in the definitions of number terms.

> Two, for instance, denotes all pairs of things, and twelve all dozens of things, connoting what makes them pairs, or dozens; and that which makes them so is something physical; since it cannot be denied that two apples are physically distinguishable from three apples, two horses from one horse, and so forth: that they are a different visible and tangible phenomenon. (*CW* 7, p. 610)

Importantly, because number terms denote physical phenomena, Mill thought that all mathematical inference requires the additional *hypothesis* of identity – i.e., that *things* behave like *units* – "that 1=1; that all numbers are numbers of the same or equal units" even though Mill concedes that this is never, in fact, true (*CW* 7, pp. 258, 259).

Mathematical propositions are thus real, not verbal, and assert empirical claims about the physical world. An arithmetical proposition, for example, "affirms that a certain aggregate might have been formed by putting together certain other aggregates, or by withdrawing certain portions of some aggregate; and that, by consequence, we might reproduce those aggregates from it, by reversing the process" (*CW* 7, p. 611).

Arithmetic operates as though it were a deductive science because of the universal generality of its basic axioms. According to Mill, these are only two: the transitivity of identity (*CW* 7, p. 610) and the addition and subtraction postulates (*CW* 7, p. 258). The remaining basic mathematical truths, he claims, derive from these by *reductio* (*CW* 7, p. 258). Like particular arithmetical truths, these basic axioms are real propositions, known by induction. For example, in considering the epistemic ground for the axiom *the sum of equals are equals* (which he considered equivalent to the metaphysical principle *Whatever is made up of parts, is made up of the parts of those parts*), Mill wrote:

> This truth, obvious to the senses in all cases which can be fairly referred to their decision, and so general as to be coextensive with nature itself, being true of all sorts of phenomena, (for all admit of being numbered,) must be considered an inductive truth, or law of nature, of the highest order. And every arithmetical operation is an application of this law, or of other laws capable of being deduced from it. This is our warrant for all calculations. (*CW* 7, p. 613)

Similarly, Mill claimed, "Every theorem in geometry is a law of external nature, and might have been ascertained by generalizing from observation and experiment…" (*CW* 7, p. 616).

Because of this, mathematics involves real inferences: "there is in every step of an arithmetical or algebraical calculation a real induction, a real inference of facts from facts; and that what disguises the induction is simply its comprehensive nature, and the consequent extreme generality of the language" (*CW* 7, p. 254).

While this empirical view that mathematical truths have the same status and foundation of laws of nature has the disquieting consequence that mathematical truths are only justifiable, and are therefore refutable, *a posteriori*, Kitcher (1998, p. 69) notes that it confers the significant benefit of explaining what he calls the "mathematical structure of reality." Nature behaves arithmetically because arithmetic describes "permanent possibilities of rearrangement" (*ibid*.).

Psychologism in Mill[15]

That Mill held both logic and mathematics to be synthetic and *a posteriori* is beyond dispute. By contrast, Mill scholarship remains divided on the point of whether his account is psychologistic. Minimally, psychologistic accounts of some domain assert that the truths of that (ostensibly non-psychological) domain are dependent upon, if not completely determined by, facts about human psychology. One recognizable version of psychologism holds that logical laws are a subset of psychological laws, and thus that the foundation of logic is ineliminably psychological. Such a position might result from the metaphysical view that reasoning and inference are psychological processes, and that in studying them the subject matter of logic is itself psychological.

In accepting Whately's definition of logic as the art and science of reasoning, and in understanding reasoning as a mental process (*CW* 7, p. 4), Mill seems committed to the psychologistic view that the subject matter of logic is psychological in nature. "Logic," Mill tells us, " ... is the science of the operations of the understanding which are subservient to the estimation of evidence: both the process itself of advancing from known truths to unknown, and all other intellectual operations in so far as auxiliary to this" (*CW* 7, p. 12). Yet, while its subject matter is psychological, logic as an art has a prescriptive, rather than a descriptive, relationship to mental processes. "Logic is not the science of belief, but the science of proof, or evidence. Insofar as belief professes to be founded on proof, the office of logic is to supply a test for ascertaining whether or not the belief is well grounded" (*CW* 7, p. 9).

Perhaps then, the prescriptive norms of logic do not depend on psychology. Yet, instead Mill seems to hold that the art of reasoning is grounded in the science of it. For Mill, the art of logic provides "the rules, grounded on that analysis [of the mental process which takes place whenever we reason] for conducting the process correctly" (*CW* 7, p. 4). Indeed, in the *Examination* (1865/1867) Mill writes that the science of reasoning is required for the *justification* logical rules:

> I conceive it to be true that logic is not the theory of thought as thought, but of valid thought; not of thinking, but of correct thinking. It is not a science distinct from, and coordinate with, psychology. So far as it is a science at all, it is a part, or branch, of psychology; differing from it, on the one hand as a part differs from the whole, and on the other, as an art differs from a science. Its theoretic grounds are wholly borrowed from psychology, and include as much of that science as is required to justify the rules of the art. (*CW* 9, p. 359)

[15] For survey of psychologism in nineteenth century logic see Brockhaus (1991) and Kusch (1995). For an extended discussion of psychologism in Mill see Godden (2005).

For example, Mill describes the principle of non-contradiction as being founded on the introspective observations concerning psychological facts.

> I consider it to be, like other axioms, one of the first and most familiar generalizations from experience. The original foundation of it I take to be, that belief and disbelief are two different mental states, excluding one another. (*CW* 7, p. 233)

It would seem, then, that, even when understood as the science of proof or evidence, psychological facts have a role not only in describing the subject matter of logic, but in formulating, grounding, and justifying logical norms and principles.

Frege's arguments against Mill's psychologism

Regardless of whether we now find Mill's account to be psychologistic or merely empiricist, a seminal criticism of this account, due to Gottlob Frege (1848-1925), identifies it as psychologistic. Frege specifically addressed Mill's account of number in his 1884 *Grundlagen* (*Foundations of Arithmetic*, 1980), and later in his 1894 review of Edmund Husserl's *Philosophy of Arithmetic* (1972).

Some of the concerns motivating Frege's attack apply even if Mill's account is only empiricist. Even this would have the unpalatable consequence that arithmetical truths would be knowable only *a posteriori* and their justification would necessarily involve contingent, physical facts. Yet Frege also found that Mill's account makes the truths of arithmetic dependent upon psychological facts such as perception and abstraction.

While Frege recognized that simple arithmetical theorems (e.g., 7+5=12) and laws (e.g., the associativity of addition) are "amply established by the countless applications made of them every day" (1980, p. 2), his logicist position claims that such observations play no part in the demonstration of such laws and truths, which are instead established by definitions and derivation from logical truths.[16] Similarly, definitions of arithmetical terms do not involve reference to facts – numbers are *not* numbers *of things*, but are instead concepts.

Frege's criticisms of Mill's "gingerbread or pebble arithmetic" (1980 p. vii) are frequently caustic and dismissive. He accused Mill of holding a "naïve" conception of number, according to which numbers are either aggregates of things, or properties of aggregates.

> The most naïve opinion is that according to which a number is something like a heap, a swarm in which the things are contained lock, stock and barrel. Next comes the conception of a number as a property of a heap, aggregate, or whatever else one might call it. Thereby one feels the need for cleansing the objects of their particularities. The present attempt belongs to those which undertake this cleansing in the psychological washtub. (1972, p. 323)

[16] As Goldfarb writes "Clearly, we would not be able to arrive at correct mathematical arguments if our inkblots were constantly to change. Yet that does not imply that mathematics presupposes the physical laws of inkblots, that those laws would figure in the justifications of mathematical laws" (2001, p. 34).

Each naïve opinion, according to Frege, yields incoherencies and false consequences, as well as confuses the origin of a concept in the mind with its meaning (1980, p. vii) and the application of a proposition or concept with its meaning (1980, p. 13).

Naïve conceptions of number are wrecked on the three reefs, Frege claimed: (i) the identity and distinguishability of numerical units, (ii) the numbers zero and one, and (iii) large numbers (1972, p. 330). Though the second and third reefs lie downstream of the first, it is convenient to start with them. The general problem is that no observed facts or entities (whether psychological or physical) seem to correspond to numbers or arithmetical truths. Frege asserted that "no one ... has ever seen or touched o pebbles" (1980, p. 11), and he wonders "[what observed or physical fact] is asserted in the definition of the number 777864" (1980, p. 9). Worse still, "If numbers are presentations, then the limited nature of our powers of presentation must also carry along with it a limitation of the domain of numbers" (1972, p. 334). As to the thesis that the truths of addition are contingent on the way objects combine, Frege chastised: "What a mercy, then, that not everything in the world is nailed down; for if it were we should not be able to bring off this separation, and 2 + 1 would not be 3" (*ibid*.). Frege further rejected Mill's contention that the identity 1 = 1 could be false "on the ground that one pound weight does not always weigh precisely the same as another;" consequently, whatever the truth-makers for propositions like'1 = 1' are, they are not (combinations of) things in the world (1980, p. 13).

The primary wrecking-point of naïve conceptions of number, Frege claimed, is their failure to comprehend identity and difference. This failure yields a convoluted account of numerical units, and hence of the number one and indeed any individual number. In trying to comprehend a naïve justification of the truth "1 + 1 + 1 = 3," Frege wrote:

> If we try to produce the number [three] by putting together distinct objects, the result is an agglomeration in which the objects contained remain still in possession of precisely those properties which serve to distinguish them from one another; and that is not the number. But if we try to do it the other way, by putting together identicals, the result runs perpetually together into one and we never reach a plurality. (1980, p. 50)

Psychological logicians attempt to overcome this difficulty by employing the mind to abstract away the properties of concrete objects which distinguish them. Yet, Frege held, such an attempt not only makes the truths of arithmetic dependent on the success of cognitive operations, it entirely fails to solve the problem since the partially-abstracted particulars will remain either distinguishable (and hence not like numerical units) or identical (and hence not combinable) (1972, p. 325 ff.).[17] "Inattention," Frege ridiculed,

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[17] See Kessler (1980) for an argument which draws upon contemporary, formalized mereology to show that Frege's criticisms can be overcome in a way consistent with Mill's basic empiricism. Roughly on Kessler's model, rather than conceive of a number as a property of an aggregate, "a number is to be understood as a special sort of relation which holds between aggregates and properties that pick out parts of those aggregates" (p. 69).

Sympathetically, Kitcher (1998, pp. 64 ff.) contends that treating numbers as mereological sums rather than as abstract properties is not only consistent with Mill's nominalism and his view that "attributions of number to agglomerations always carry with them some way in which the agglomeration is supposed to be

"... [has become] an exceedingly effective logical power; whence, presumably, the absentmindedness of scholars" (1972, 324).

Despite these differences, Frege and Mill shared what Goldfarb (2001, p. 28) has called a *universalist conception* of logic, whereby logical laws are universal truths (cf. Skorupski 1998, p. 44). For Mill, logical truths have the same scope and justification as laws of nature, while for Frege their scope is much broader and their justification categorically different. For Frege, the laws of logic are the laws of Thought – "laws of the laws of nature" – which hold between judgments some of which express the laws of nature (1980, p. 99).

Conclusion

Importantly, just as developments in logical techniques within and following the time of Mill's *Logic* rendered its account of the structure of deduction antiquated, a similar fate affected its empiricist meta-logic and account of the deductive sciences. As Goldfarb (2001, pp. 26 ff.) describes, the universalist conception of logic shared by Mill and Frege was replaced with the contemporary, Tarski-Quine schematic conception. On the schematic conception, logic is not about the world – neither the world of concrete particulars nor the world of abstract universals. Rather, "the subject matter of logic consists of logical properties of sentences and logical relations among sentences" (Goldfarb 2001, p. 26). Logic, on this conception, is a properly formal science. Its formuale are schemata, or logical forms, which are representations of the structural (e.g., truth-functional, quantificational, relational) composition of sentences using logical signs. Schemata are not about anything; they have no semantic content and hence are neither true nor false. When interpreted, the elements of the schemata are supplied with a subject matter, or semantic content; interpreted formulae thereby receive a truth-value. "[O]ne schema *implies* another, that is, the second schema is a *logical consequence* of the first, if and only if every interpretation that makes the first schema true also makes the second true" (*ibid*.). Although Mill never conceived of logic as a purely formal science, it would likely not have been satisfactory to him, since he required that logic contain both real propositions and real inferences if it was to be capable of advancing knowledge.

In summary, this chapter has offered an overview of Mill's *System of Logic* (1843/1872) by situating its development in the context of some of the meta-logical themes and disputes characteristic of the 19th century as well as his empiricism. Mill's early views on the nature and utility of syllogistic reasoning were significantly shaped by Whately's (1827) *Elements of Logic*. While aspects of the *Elements* remain in the *Logic*, Mill, came to hold that not only must induction be a genuine form of inference but indeed that "all [real] inference is from particulars to particulars." Having been inspired by Stewart, Mill concluded that the only account of the foundation ratiocination consistent with his own empiricism and capable of solving the "great paradox" of the informativeness of deduction was that ratiocination depends on the cogency of induction since all general claims, including the major premises of all syllogisms, are "registers of inferences already made." The proper task of the inductive logician is no different than

divided up into parts" (cf. *CW* 7, p. 610), but that it also avoids Frege's central criticism that naturalized accounts of number fail to comprehend the identity and difference of numerical units.

that of the natural scientist, namely the discovery of the true causal laws of the universe. These, contrary to Whewell, are properly understood to be found "out there," among the phenomena, and are not supplied by the mind through colligation. Only through the knowledge of causes can one know which inferences from the particular and known to the general and unknown are legitimate because the relation in question rightly instantiates some causal law. This empiricist approach to the epistemic foundations of logic led Mill to offer a naturalized account of putatively *a priori* disciplines such as the science of number, which in turn attracted Frege's criticisms of the *Logic* as psychologistic.

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